Multiplexer Channel-Separating Units Using Interdigital and Parallel-Coupled Filters

G. L. MATTHAEI, FELLOW, IEEE, AND E. G. CRISTAL, MEMBER, IEEE

Abstract—**Design theory is presented for strip-line multiplexer channel-separating units which have constant-resistance input impedances so that many units can be cascaded without reflection effects. Each channel unit consists of an interdigital band-pass filter and a band-stop filter which uses parallel-coupled resonators. In order to obtain the desired constant-resistance input impedance, both filters are designed from singly loaded, maximally flat, low-pass prototype filters. A trial design was worked out and constructed so as to have five per cent bandwidth for the separated channel. Excellent agreement between theory and experiment was obtained.**

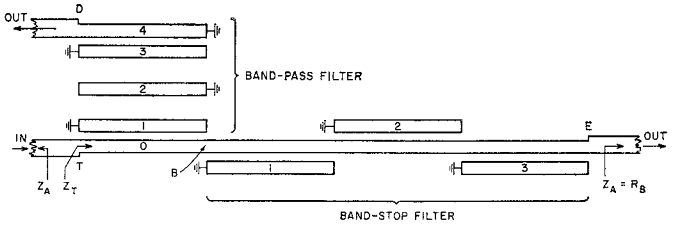

Fig. 1. A strip-line multiplexer channel unit.

I. Introduction

THE DESIGN OF multiplexers, which separate out frequencies in certain ranges from a spectrum of signals covering a large range of frequencies, is a problem frequently encountered in microwave engineering. The separation of the desired frequency bands can be accomplished by use of band-pass filters, but special design procedures are required so that the filters can be interconnected without undesirable interaction effects. In this paper a design technique is described that makes use of channel-separating units, each of which consists of a band-pass filter along with a band-stop filter. In theory, these channel units present a constant-resistance input impedance, and as a result a number of units designed to separate out a number of frequency bands can be cascaded without harmful interaction effects [1], [2], [3]. The filter structures used have the additional advantage of being relatively easy to fabricate.

A sketch of the strip-line multiplexer channel unit is given in Fig. 1. It consists of an interdigital band-pass filter and a parallel-coupled-resonator band-stop filter. In Fig. 1, Z_A represents the generator and terminating resistance. The input impedance Z_T is, in theory, exactly equal to Z_A at all frequencies. Thus the load impedance Z_A may be replaced by another channel unit, and a number of channel units may be cascaded.

The basic method for designing a reflectionless multiplexer channel unit uses a singly loaded (i.e., with a resistor termination at one end only) low-pass prototype filter in the design of both the band-pass filter and the band-stop filter. The low-pass prototypes for both filters should have the same number of elements and should ideally be maximally flat in the pass band,[1] in order for the derived band-pass and band-stop filter to be complementary filters (that is, filters having input impedances whose imaginary parts cancel and whose real parts add to give a constant). Herein the two filters are connected in series, giving an input impedance for the channel unit which is a constant resistance equal to the generator resistance. A proof for this constant-resistance input-impedance property is given in the Appendix.

II. Synthesis Method for the Band-Stop Filter Used in the Channel Unit

The synthesis procedure for the realization of the band-stop filter of the channel unit is basically the exact synthesis method of Ozaki and Ishii [4], and of Schiffman and Matthaei [5]. However, it is fundamental to the design theory presented in this paper that an alternate method of applying Kuroda's Identity to that previously given [5], [6], be used in the synthesis of the band-stop filter of the channel unit.

The reasons for using an alternate method of applying Kuroda's Identity are as follows: The band-stop filter design equations of Schiffman and Matthaei [5] were derived by starting from a lumped-element low-pass prototype filter, then converting this low-pass filter into a band-stop filter by use of a tangent-function mapping. The mapping function converted shunt capacitors into shunt, open-circuited stubs, and series inductances into series, short-circuited stubs. The input impedance of this filter had the properties desired for use in multiplexer channel units, but the series stubs in the structure were impractical to build. Kuroda's Identity [4], [5], [6] was then used to feed sections of line into the structure from both ends, so as to yield a modified structure having the form of shunt stubs separated

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G. L. Matthaei is with the Dept. of Electrical Engineering, University of California, Santa Barbara, Calif. He was formerly with the Stanford Research Inst., Menlo Park, Calif.

E. G. Cristal is with the Stanford Research Inst., Menlo Park, Calif.

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[1] It would also be satisfactory for many applications to use singly loaded filters with small Chebyshev pass band ripples. This would, however, cause the input impedance to deviate from the desired constant resistance.

by sections of lines (the stubs and lines all being a quarter-wavelength long at mid-stop-band). Feeding the extra sections of line in from both ends of the filter had the effect of altering the input impedance characteristic at both ends of the filter. In the case of a band-stop filter having a sizeable stop-band width, these altered input impedances would have added series- and shunt-resonance effects that would disrupt the performance of the band-pass filter in the multiplexer channel unit.

In order to eliminate this problem, an alternate method of applying Kuroda's Identity was proposed. In the alternate method, the Identity is applied from the load end only (i.e., from the end away from the band-pass filter). In this way the input impedance Z_s of the band-stop filter is a simple tangent-function mapping of the input impedance of the prototype filter. That is, if $Z(\omega')$ is the input impedance of the prototype filter, then $Z_s = Z[\Lambda \tan (\pi\omega/2\omega_0)]$ is the input impedance of the derived band-stop filter as indicated in Fig. 2. The symbols used as the argument of the impedance function are defined below:

ω' is the frequency variable of the low-pass prototype filter.
ω_1' is the cutoff frequency of the low-pass prototype filter.
ω is the frequency variable of the band-stop filter.
ω_0 is the center frequency of the band-stop filter.
Λ is a scaling parameter that is defined by

$$\Lambda = \omega_1' \tan [(\pi/4)w]. \tag{1}$$

The symbol w in (1) is the fractional stop-band width of the band-stop filter. It is defined by

$$w = \frac{\omega_2 - \omega_1}{\omega_0} \tag{2}$$

where ω_2 and ω_1 are the upper and lower cutoff frequencies of the band-stop filter corresponding to ω_1' of the low-pass prototype filter.

By using this alternate method, both the band-pass and band-stop filter may be synthesized from singly loaded low-pass prototype filters, which should permit close realization of the desired theoretical performance. The alternate method of applying Kuroda's Identity is shown schematically in Fig. 2. The example in Fig. 2 uses a singly loaded prototype, since the primary use of the alternate method will probably be in its application to design of multiplexer channel units. However, this technique can also be used for doubly loaded prototypes. When the alternate method of applying Kuroda's Identity is used, the previously presented exact synthesis design formulas [5] and tables of h parameters [6] do not apply. However, the algorithm [6] for computing the h parameters of band-stop filters remains valid and can be easily adapted to include the case where Kuroda's Identity is applied from the load end only. Only two modifications to the algorithm are necessary in order to use this alternate design method:

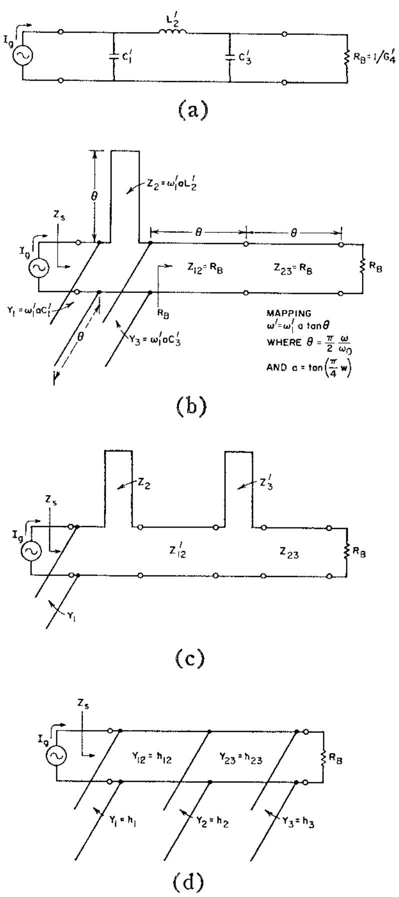

Fig. 2. Stages in the transformation of a singly loaded, low-pass, prototype filter into a band-stop, transmission-line filter. (a) Singly loaded prototype. (b) Mapped prototype. (c) After applying Kuroda's Identity to Y_3 and Z_{12} in (b). (d) After applying Kuroda's Identity to Z_2 and Z_{12}' and to Z_3' and Z_{23} in (c).

1) The determination of the parameter P is now given by[2]

$$P = n - 1 \tag{3}$$

for both n even and odd, where n is the number of reactive elements in the low-pass prototype filter.

2) $$h_1 = g_1'. \tag{4}$$

All other details of applying the algorithm are as previously described [6].

An example will now be given of the calculation of the h values of a three-resonator, 0.05-fractional-band-width, band-stop filter based on a maximally flat prototype [7], [8].[3] The results will be used in the synthesis of a trial channel unit in strip line which is described in Section IV.

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[2] See Cristal [6], p 370, for the significance of the parameter P.

[3] The g_i values for singly loaded low-pass prototype filters given in [7] and [8] are numbered so that the resistive load element is given by g_0, and the adjacent reactive element is then denoted by g_1, and so on. In order to use these element values in the algorithm, and in order to make the notation conform to that in Fig. 2(a), the subscripts of the g's in [7] and [8] should be altered according to the rule

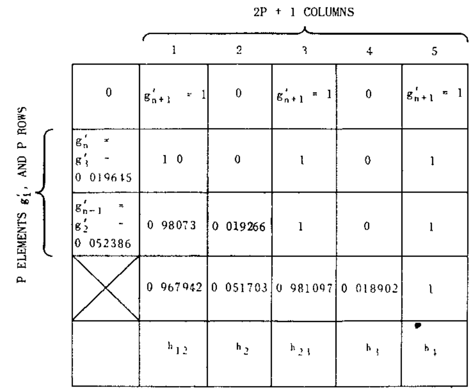
$$g_i = g_{n+1-i}$$

where n is the number of reactive elements in the low-pass prototype filter.

TABLE I

g_i AND g_i' VALUES FOR A MAXIMALLY FLAT SINGLY LOADED LOW-PASS PROTOTYPE FILTER [7], [8]

$g_0 = \infty$	$g_1 = 1.500$	$g_2 = 1.333$	$g_3 = 0.500$	$g_4 = (\text{load}) = 1.0$
$g_0' = \infty$	$g_1' = 0.058935$	$g_2' = 0.052386$	$g_3' = 0.019645$	$g_4' = 1.0$



	1	2	3	4	5
0	$g'_{n+1} = 1$	0	$g'_{n+1} = 1$	0	$g'_{n+1} = 1$
$g'_n = g'_3 = 0\ 019645$	1 0	0	1	0	1
$g'_{n-1} = g'_2 = 0\ 052386$	0 98073	0 019266	1	0	1
	0 967942	0 051703	0 981097	0 018902	1
	h_{12}	h_2	h_{23}	h_3	h_1

Fig. 3. Computation matrix for h parameters of a singly loaded band-stop filter.

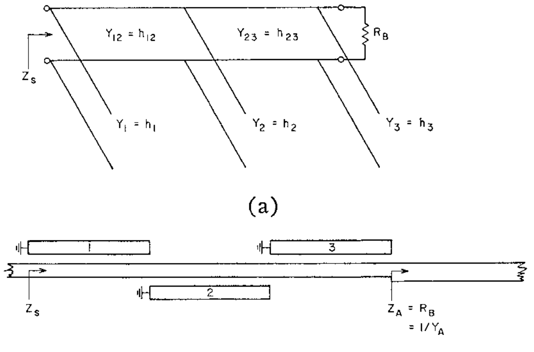


Fig. 4. (a) Open-wire-line, band-stop filter. (b) Its parallel-coupled strip-line equivalent.

The g and g' values used in the example calculations are given in Table I. Recall that g and g' are related by [6]

$$g_i' = g_i\omega_1' \tan\left(\frac{\pi}{4} w\right). \tag{5}$$

The parameter P, given by (3), is

$$P = n - 1 = 3 - 1 = 2. \tag{6}$$

The computation matrix [6] is shown in Fig. 3. The parameter h_1 is given according to (4) by g_1' and is therefore equal to 0.05894.

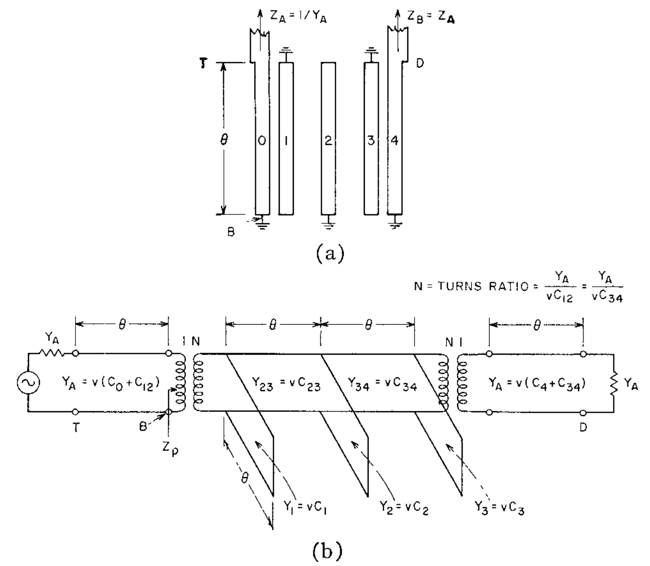
The transmission-line filter corresponding to the h values given in Fig. 3 is shown in Fig. 4(a), and its exact realization as a parallel-coupled-resonator strip-line filter is given in Fig. 4(b). Design equations that relate the admittances of the transmission lines and stubs of the filter shown in Fig. 4(a) to the normalized self and mutual distributed capacitances of the coupled resonators in Fig. 4(b) are given in [5]. From these equations the band-stop filter may be constructed [5].

III. THE BAND-PASS FILTER AND THE INTERCONNECTION

Figure 5(a) shows an interdigital band-pass filter such as is used in the multiplexer channel unit in Fig. 1. Equations for the design of filters of this type from low-pass prototype filters are presented in Matthaei [9]. Note that the interdigital filter structure in Fig. 5(a) is of the type that uses short-circuited input and output lines, which act as impedance transforming sections (not as resonators) [9]. Thus, the structure shown is a three-resonator filter (any number of resonators may be used, of course), the resonators being formed from Lines 1, 2, and 3. This form of interdigital band-pass filter, which uses short-circuited input and output lines, is most practical for narrow to moderate bandwidths [9].

Figure 5(b) shows an equivalent circuit for the filter in Fig. 5(a). Though this equivalent circuit involves some approximations [9] it has been found to give very good accuracy in representing the performance of interdigital filters of the form shown in Fig. 5(a). Two parts of this equivalent circuit are not evident in Matthaei [9]. These are the lengths of transmission line of length θ and characteristic admittance Y_A at each end of the filter. The exact equivalent circuit in Fig. 21 of Matthaei [9] should have included such a length of line between the ideal transformer and the load conductance Y_A at the left in the open-wire circuit, in order for the open-wire equivalent circuit to be exactly equivalent to the parallel-coupled-strip circuit in all respects. However, the discussion in Matthaei [9] was only concerned with the *input admittance* seen looking in from the right, so the presence of this section of matched transmission line was of no importance for that analysis. But for the present situation, the presence of this length of line is of importance in understanding the operation of the multiplexer channel unit. [Note that points T and D in Fig. 5(a) correspond to points T and D indicated in Fig. 5(b).]

In order to use a filter of the form in Fig. 5(a) in a band-pass-plus-band-stop filter connection such as that in Fig. 1, it is necessary to break the ground connection on line 0 at point B indicated in Fig. 5(a). By study of various parallel-coupled line configurations and their known, exact, open-wire equivalent circuits [10], it was determined that at least to an excellent approximation (and probably as an exact equivalence), breaking the circuit in Fig. 5(a) at point B, and inserting an added impedance between the end of line 0 and ground, has the effect of breaking the circuit in Fig. 5(b) at the indi-



(a)

(b)

Fig. 5. An interdigital filter and its open-wire-line equivalent circuit. The capacitance C_i is the capacitance per unit length between strip i and ground, and capacitance $C_{i,i+1}$ is the capacitance per unit length between strips i and $i+1$. The transformers shown are ideal transformers, and v is the velocity of propagation.

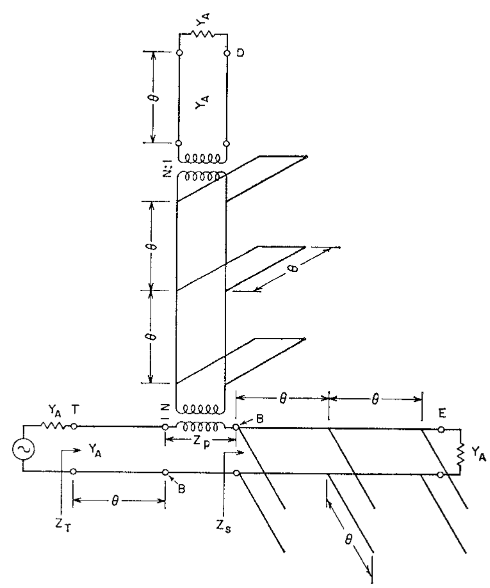


Fig. 6. Open-wire equivalent circuit for the multiplexer channel unit in Fig. 1.

cated point B, and inserting the added impedance in series with the ideal transformer.

Making use of the equivalent circuits in Figs. 4(a) and 5(b), along with the foregoing observation, we see that the multiplexer channel unit in Fig. 1 has the open-wire equivalent circuit shown in Fig. 6. Note that in this circuit the connection at point B in Fig. 5(b) has been broken, and the input terminals of the band-stop filter circuit have been connected so as to reclose the circuit. Note that looking left from the left half of the bisected Point B, a constant-resistance Z_A will be seen since the transmission line of characteristic impedance Z_A is terminated in a matching generator impedance Z_A. The ideal transformer at the input of the band-pass filter has an impedance-level-transforming effect, but otherwise has no effect on the character of the input impedance to the band-pass filter. Thus, the impedance Z_p associated with the band-pass filter is of the form typical of a band-pass filter that starts out with a shunt, band-pass resonator. The band-stop filter starts out with a shunt, band-stop resonator, which makes the two filters of the proper form for series interconnection. (Note that if the band-stop filter started out with a series band-stop resonator branch, it would present an open circuit at resonance, which would disrupt the performance of the band-pass filter.)

If the band-stop filter is designed from a singly loaded, maximally flat low-pass prototype filter using the methods described in Section II, and if the band-pass filter is designed using the same low-pass prototype and for the same 3-dB bandwidth as the band-stop filter, then the input impedances Z_p and Z_s will be complementary. This means that the imaginary parts of these impedances will cancel, while their real parts will add up to a constant such that

$$Z_p + Z_s = Z_A. \qquad (7)$$

Thus, as has been previously mentioned, it is possible (at least to a good approximation) for the input impedance of multiplexer units of the form in Fig. 1 to have a perfect impedance match with respect to the line impedance Z_A at all frequencies.

The perfect impedance match described above is contingent upon the use of singly loaded, maximally flat prototype filters. However, in some cases it may be desirable to use singly loaded, Chebyshev low-pass prototype filters instead, as a basis for design. In this case the input VSWR of the multiplexer channel unit would not be as good (though it would be acceptable for most applications), but the filters would have the advantage of more uniform pass-band attenuation and sharper rate of cutoff, as is typical of Chebyshev filters in comparison to corresponding maximally flat filters. If Chebyshev filters are used, they should also be designed so that the band-stop and band-pass filters have the same 3-dB bandwidth.

The design equations in Table II of Matthaei [9] can be used without any changes, for the design of band-pass interdigital filters of the form in Fig. 1. However, some additional comments are required with respect to the use of the singly loaded low-pass prototype element values in Matthaei, et al. [7], (and similar comments apply with respect to those in Weinberg [8]). First, the element values in Matthaei, et al. [7], are numbered, so that the resistive load element is $g_0 = 1$, the adjacent reactive element is g_1, and the element values are numbered consecutively up to g_n, which is the reactive element adjacent to the generator; and $g_{n+1} = \infty$ is the internal impedance (or, for the dual case, internal admittance) of the generator. To correspond to the notation in Fig. 5 and in Matthaei [9], we wish to reverse the order of numbering all these element values so that $g_0 = \infty$, and so on consecutively to $g_{n+1} = 1$. Now for

TABLE II

DIMENSIONING OF FILTERS IN CHANNEL UNIT

(a) Band-pass filter rod diameters and center to center spacing in inches

k	Diameter of Rod k	k	Center to Center Spacing ($c_{k,k+1}$) of Rods k and $k+1$
0	0.331	0	0.486
1	0.217	1	0.792
2	0.224		
3	0.203	2	0.674
4	0.318	3	0.384

(b) Band-stop filter rod diameters and center to center spacing in inches

k	Diameter of Rod k		k	Center to Center Spacing ($c_{k,k}$) of k_a Rod and k_b Rod
	k_a Rod	k_b Rod		
1	0.332	0.215	1	0.663
2	0.336	0.219	2	0.686
3	0.344	0.237	3	0.846

Ground Plane Spacing 0.625

computing an interdigital filter design such as that in Fig. 5, we want the impedance Z_p looking into the filter to be that of a filter designed so that it would give a prescribed maximally flat (or Chebyshev) response if driven by an infinite-internal-impedance current generator. However, in this application we are not actually going to drive the filter from a current generator, but instead, from a source of impedance Z_A. For this reason we must change the prototype driving source impedance from $g_0=\infty$ to $g_0=g_{n+1}=1$. After reversing the order of numbering of the element values as described, and then setting $g_0=g_{n+1}=1$, the prototype element values can be used directly in Table II of Matthaei [9], and the band-pass filter design worked out as described in that paper. As is indicated in Fig. 1, line 0 in Fig. 5(a) becomes part of the main transmission line in the completed multiplexer channel unit.

IV. TRIAL MULTIPLEXER CHANNEL UNIT IN STRIP LINE

Based on the theory and design techniques presented in Sections II and III a multiplexer channel unit was constructed in strip line. The filters of the channel unit were based on a three-resonator, singly loaded, maximally flat low-pass prototype. The design center frequency was 1.5 Gc, and the design fractional bandwidth was 0.05. The interdigital band-pass filter and the parallel-coupled-resonator band-stop filter were both constructed using round rods between parallel ground planes [11].

A drawing of the channel unit, showing important dimensions of the filters, is given in Fig. 7, while Table II gives additional dimensions. A photograph of the constructed channel unit with its top ground plane removed is shown in Fig. 8.

The band-stop filter was tuned by individually resonating each of the band-stop filter resonators while the remaining two were detuned. Resonance was determined by tuning for minimum transmission from Port 1 to Port 3. Initial tuning of the band-pass filter was accomplished using the alternating short-circuit and open-circuit procedure [12], [13]. Fine tuning adjustments were made by observing the reflected wave of the channel unit using an electronically swept frequency source, a reflectometer, and an oscilloscope.

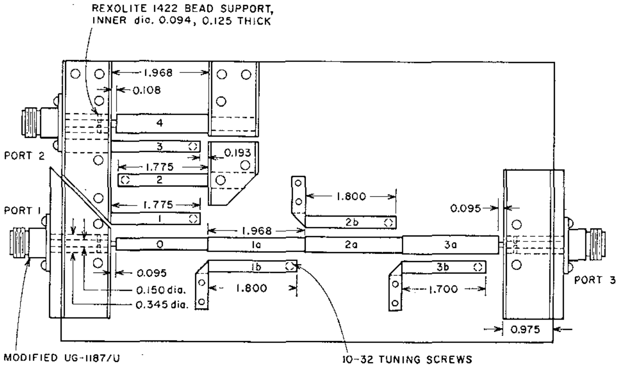


Fig. 7. Drawing of the trial multiplexer channel unit in strip line.

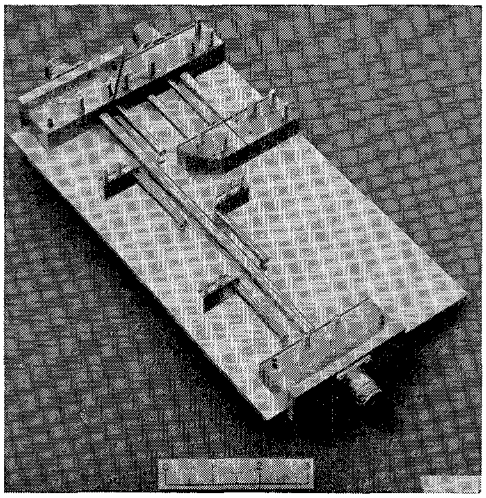


Fig. 8. Photograph of the trial multiplexer channel unit with its cover plate removed.

The VSWR of the channel unit (at Port 1) from 1 to 4 Gc is given in Fig. 9. An expanded frequency scale is also shown in this figure, giving the VSWR in the vicinity of the pass band of the interdigital filter. It is seen to be better than 1.25 everywhere in the 1-to-4-Gc interval. The sinusoidal variation of VSWR seen in Fig. 9 is due primarily to the junction effect of the transitions associated with the type-N connector at Ports 1 and 3, although there is some contribution to the VSWR from the connectors themselves as well as from the termination at Port 3. The maximum magnitude of the sinusoidal variation of VSWR suggests that the VSWR of the transition from coaxial line to the round conductor

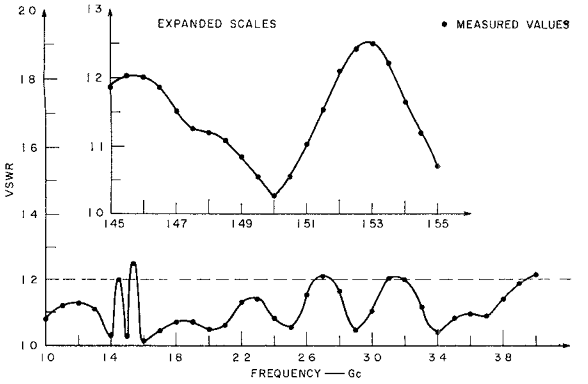


Fig. 9. Measured VSWR of a trial channel unit in strip line.

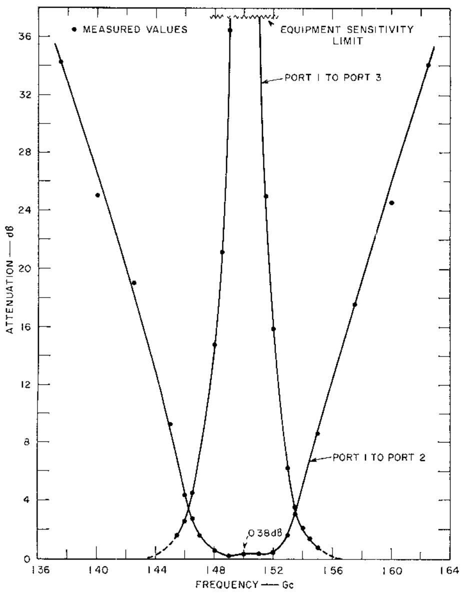


Fig. 10. Measured attenuation characteristics of a trial channel unit in strip line.

between parallel ground planes is approximately 1.1 per transition. This could be reduced by further refining the transitions. However, the measured results adequately demonstrate the design principles that were to be shown.

The measured attenuations from Port 1 to Port 3 and from Port 1 to Port 2 in the vicinity of the pass band of the band-pass filter are given in Fig. 10. The attenuation at the band-center frequency, 1.5 Gc, is 0.38 dB. This value compares favorably with that given by a first-order formula for the attenuation of the band-pass filter of a channel unit at center frequency:

$$(L_A)_0 \approx \frac{4.34\omega_1'}{w}\left[\left(\sum_{i=1}^{n}\frac{g_i}{Q_{u_i}}\right)+\frac{1}{\hat{Q}_{u_1}\hat{g}_1}\right] \text{dB} \qquad (8)$$

where

$(L_A)_0$ is the attenuation in dB at the center frequency

w is the fractional bandwidth

ω_1' is the cutoff frequency of the low-pass prototype filter

Q_{u_i} is the unloaded Q of the ith resonator of the band-pass filter

$\hat{Q}_{u_i}$ is the Q of the first resonator of the band-stop filter

g_i is the ith element value of the low-pass prototype filter used for designing the band-pass filter

$\hat{g}_1$ is the g_1 element value of the low-pass prototype filter used for designing the band-stop filter.

Using the same unloaded Q of 1000^4 for each of the resonators and (8) gives a calculated value of 0.347 dB for L_{A_0}.

From Fig. 10, it is seen that the crossover frequencies occur virtually at the 3-dB points, and that the 3-dB fractional bandwidth is

$$2\left(\frac{1.536-1.463}{1.536+1.463}\right)=0.0488,$$

which is in good agreement with the design value of 0.05.

V. Conclusions

The multiplexer channel unit in strip line was found to have a VSWR and an attenuation response that corresponded excellently with that predicted by the theory. The off-band VSWR was sufficiently small (and could be made smaller by improving the design of the transitions from the type-N connectors to the round conductor between parallel ground planes) to enable several channel units to be cascaded without undue interaction effects. Use of the new method of designing the band-stop filter of the channel unit resulted in a measured bandwidth of both filters that was accurately predictable from the theory. Aligning of the channel unit was straightforward and is not made appreciably more complicated by adding more resonators.

Appendix

Proof that Series-Connected Maximally Flat, Singly Loaded Filters Can Give a Constant-Resistance Input Impedance

Let us first consider the case of lumped-element LC low-pass and high-pass filters such as might be used in a lumped-element diplexer. A singly terminated, maximally flat, low-pass filter having a one-ohm resistor at one end and an infinite-internal-impedance current generator (of current I_g) at the other end would deliver power P to the resistor in accordance with

$$P = |I_g|^2 \operatorname{Re} Z_L \qquad (8)$$

where

$$\operatorname{Re} Z_L = \frac{1}{1+\omega^{2n}}, \qquad (9)$$

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[4] A Q_u of 1000 represents a typical value of unloaded Q for strip-line resonators of the type used.

Z_L is the input impedance seen from the current generator, ω is the radian frequency, and n is the number of reactive elements in the filter. Thus, we see that Re Z_L gives the power transfer function, and the 3-dB point occurs where $\omega=1$ (and Re $Z_L=0.5$). To obtain Re Z_H for a corresponding high-pass filter having a 3-dB point at $\omega=1$, we map (9) by replacing ω by $1/\omega$ which gives

$$\text{Re } Z_H = \frac{\omega^{2n}}{1+\omega^{2n}} \cdot \tag{10}$$

If these two filters are connected in series, the real part of the total input impedance Z_T will be

$$\text{Re } Z_T = \frac{1}{1+\omega^{2n}} + \frac{\omega^{2n}}{1+\omega^{2n}} = 1. \tag{11}$$

Thus we see that the real parts add to one at all frequencies.

The next point, which we shall briefly prove, is that if the real part of a minimum-reactance impedance (i.e., an impedance having no poles on the $j\omega$ axis) is a constant vs. frequency, then its imaginary part is zero. For reasons indicated later in the paper, Z_L and Z_H are minimum-reactance impedances, so Z_T is also a minimum-reactance impedance. Thus in view of (11),

$$Z_T = Z_L + Z_H = 1 \tag{12}$$

at all frequencies ω, and the filters are said to be complementary.

The poles of Z_L are the natural frequencies of vibration when Z_L is driven by a current generator, and for a maximally flat filter of the type under discussion the natural frequencies of vibration are well known to lie on a semicircle in the left half of the complex-frequency plane. Because there are no poles of Z_L on the $j\omega$ axis, Z_L is a minimum-reactance impedance. Z_H is also a minimum-reactance impedance for similar reasons.

To prove that Im $Z_T=0$ for all ω if Re $Z_T=$ (a constant), we invoke equation (8.05-3) of Tuttle [14], which in our notation is

$$\text{Im } Z_T(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\text{Re } Z_T(\lambda) - \text{Re } Z_T(\omega)}{\lambda - \omega}\, d\lambda. \tag{13}$$

(This relation applies only to minimum-reactance impedances.) Since Re $Z_T(\lambda) = $ Re $Z_T(\omega) = 1$ for all λ and ω, the integral must equal zero.

If a low-pass-to-band-pass transformation is applied to the filters previously discussed, the low-pass filter will become a band-pass filter, and the high-pass filter will become a band-stop filter. This mapping process would distort the frequency scale of Z_T from what it was before being mapped, but since $Z_T=1$, the mapped Z_T will still be a constant resistance. Thus singly loaded, maximally flat band-pass and band-stop filters connected in series can also be complementary and give a constant-resistance input. (Also, a similar proof on the dual basis applies to the parallel connection of filters.)

In the case of the filters in Fig. 6, the input impedance Z_s of the band-stop filter can be obtained exactly from a *high-pass*, lumped-element prototype by use of the mapping $\omega = -A \cot\theta$, where A is a constant. The band-pass filter is not obtainable *exactly* from a conventional lumped-element prototype, but if the band-pass filter is designed by the method of Matthaei [9], Z_p will be a close approximation to the impedance obtained by applying the same $\omega = -A \cot\theta$ mapping to a conventional *low-pass*, lumped-element-filter prototype. Thus if the prototypes are complementary, the mapped filters will also be complementary (at least to an excellent approximation).

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